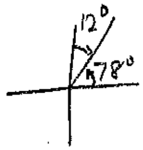
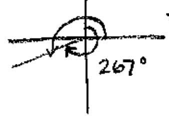
A boat sails 21 nautical miles on a bearing of 12° from port, then changes direction, and sails an additional 13 nautical miles on a bearing of 267°.

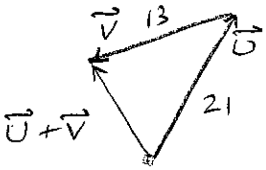
SCORE: ________ / 20 PTS

[a] Find the vector which represents the boat's final position relative to port.





$180^\circ + 270^\circ - 267^\circ$
$= 183^\circ$

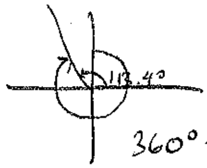


$\vec{u}+\vec{v} = 21\langle \cos 78^\circ, \sin 78^\circ\rangle + 13\langle \cos 183^\circ, \sin 183^\circ\rangle$

$\approx \langle -8.6, 19.9\rangle$

[b] What is the bearing of the boat's final position relative to port ?

$\theta_{\vec{u}+\vec{v}} = 180^\circ + \tan^{-1}\frac{19.9}{-8.6} \approx 113.4^\circ$



$360^\circ - 113.4^\circ + 90^\circ = 336.6^\circ$

BEARING $= 336.6^\circ$

[c] How far is the boat from port at the end ?

$\|\vec{u}+\vec{v}\| = \sqrt{(-8.6)^2 + 19.9^2} \approx 21.7$ NAUTICAL MILES

Suppose that $S = 72^\circ$ and $p = 28$. Find all values of s for which there are two possible triangles ΔSPA.

SCORE: ________ / 5 PTS

$p \sin S < s < p$

$28 \sin 72^\circ < s < 28$

$26.6 < s < 28$

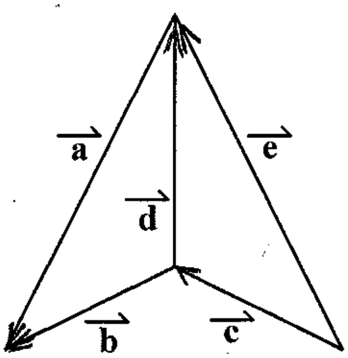
MULTIPLE CHOICE: **Circle the letter corresponding to the correct answer.** SCORE: _______ / 5 PTS

If $d = 3.2$ and $n = 1.9$, there is a possible triangle ΔDNE if $e =$

$3.2 + 1.9 > 5.0$

[A] 1.1 [B] 5.2 [C] 1.3 [D] 5.0 [E] none of the above

Write vectors $\vec{d}$ and $\vec{e}$ in terms of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ in the diagram below. SCORE: _______ / 5 PTS



$\vec{d} + \vec{a} = \vec{b} \longrightarrow \vec{d} = \vec{b} - \vec{a}$

$\vec{c} + \vec{d} = \vec{e} \longrightarrow \vec{e} = \vec{c} + \vec{b} - \vec{a}$

Solve triangle ΔFUN if $U = 64^\circ$, $\overset{u}{\cancel{f}} = 7.8$ and $f = 8.3$. **Sketch and label triangles with your final answers.** SCORE: _______ / 15 PTS
If no such triangle exists, write "DNE". If more than one triangle is possible, solve for all possible triangles.

$f \sin U = 8.3 \sin 64^\circ \approx 7.5$

$f \sin U < u < f \rightarrow 2\ \Delta\text{'s}$

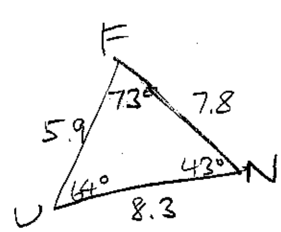
$\frac{\sin F}{8.3} = \frac{\sin 64^\circ}{7.8}$

$\sin F = \frac{8.3 \sin 64^\circ}{7.8}$

$F \approx 73.0^\circ$ or $180^\circ - 73.0^\circ = 107.0^\circ$

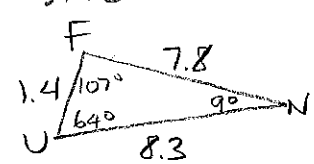
$N = 180^\circ - (64^\circ + 73^\circ)$
$= 43^\circ$

$\frac{n}{\sin 43^\circ} = \frac{7.8}{\sin 64^\circ} \longrightarrow n = \frac{7.8 \sin 43^\circ}{\sin 64^\circ} \approx 5.9$



$N = 180^\circ - (64^\circ + 107^\circ) = 9^\circ$

$\frac{n}{\sin 9^\circ} = \frac{7.8}{\sin 64^\circ} \longrightarrow n = \frac{7.8 \sin 9^\circ}{\sin 64^\circ} \approx 1.4$



Find the area of triangle ΔQED if $q = 4.1$, $d = 10.5$, $Q = 21^\circ$, $E = 46^\circ$ and $D = 113^\circ$. SCORE: _______ / 5 PTS

$\frac{1}{2}(4.1)(10.5)\sin 46^\circ \approx 15.5$

Let $\vec{l}$ be the vector $5\vec{i} - 7\vec{j}$. **SCORE: _______ / 45 PTS**

Let $\vec{g}$ be the vector $< -12, -4 >$.

[a] Find a vector of magnitude 8 in the opposite direction as $\vec{g}$. Write your final answer as a linear combination of $\vec{i}$ and $\vec{j}$.

$$-8\left(\frac{1}{\|\vec{g}\|}\right)\vec{g} = \frac{-8}{\sqrt{(-12)^2+(-4)^2}}\langle -12,-4\rangle = \frac{-8}{4\sqrt{10}}\langle -12,-4\rangle = -\frac{\sqrt{10}}{5}\langle -12,-4\rangle$$

$$= \frac{12\sqrt{10}}{5}\vec{i} + \frac{4\sqrt{10}}{5}\vec{j}$$

$$\approx 7.6\vec{i} + 2.5\vec{j}$$

[b] If $\vec{l}$ is perpendicular to $\vec{r} = a\vec{i} + (a+4)\vec{j}$, find the value of a.

$$\vec{l}\cdot\vec{r} = 0 \longrightarrow 5a - 7(a+4) = 0$$

$$-2a - 28 = 0$$

$$a = -14$$

[c] Find the projection of $-10\vec{j}$ onto $\vec{l}$.

$$\frac{-10\vec{j}\cdot\vec{l}}{\vec{l}\cdot\vec{l}}\vec{l} = \frac{70}{74}\langle 5,-7\rangle = \left\langle \frac{350}{74}, \frac{-490}{74}\right\rangle \approx \langle 4.7, -6.6\rangle$$

[d] Find the angle between $\vec{l}$ and $\vec{g}$.

$$\cos^{-1}\frac{\vec{l}\cdot\vec{g}}{\|\vec{l}\|\|\vec{g}\|} = \cos^{-1}\frac{-32}{(\sqrt{74})(4\sqrt{10})} \approx 111.7^\circ$$

[e] If $\vec{l}$ represents a force vector that is applied to an object to move it from the point $(1, -3)$ to the point $(-2, -9)$, find the work done.

$$\vec{d} = \langle -2-1, -9-^-3\rangle = \langle -3, -6\rangle$$

$$\vec{l}\cdot\vec{d} = 27$$